INTÉGRALES II

Primitives usuelles

Fonction	Primitives
0	$c \quad (c \in \mathbb{R})$
$a \quad (a \in \mathbb{R})$	$ax+c$
x^n	$\dfrac{x^{n+1}}{n+1} + c$
$\dfrac{1}{x}$	$\ln x + c$
$\ln x$	$x\ln x - x + c$

Fonction	Primitives
e^x	$e^x + c$
$\dfrac{1}{x^2}$	$-\dfrac{1}{x} + c$
$\sin x$	$-\cos x + c$
$\cos x$	$\sin x + c$
$\operatorname{tg} x$	$-\ln\lvert\cos x\rvert + c$

Intégration par parties

Formule : "mémo"

$$\int u'v = [uv] - \int uv'$$

Disposition pratique :

u'=... u =...

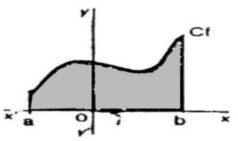
v'=... v=...

Changement de variable

ex. $I = \int_1^2 \ln(2x+3)dx$. On pose u=2x+3. Alors $dx = \frac{1}{2}du$ d'où :

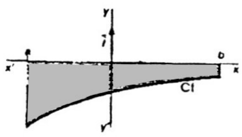
$$I = \frac{1}{2}\int_5^7 \ln u \, du$$ *(ne pas oublier de changer les bornes)*

Calculs d'aires



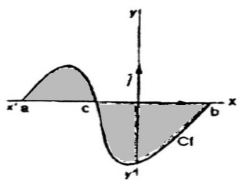
$$D = \left\{ M(x,y) \in \mathscr{P} \,/\, \begin{matrix} a \leqslant x \leqslant b \\ 0 \leqslant y \leqslant f(x) \end{matrix} \right\}$$

$$\mathscr{A}(D) = \int_a^b f(x)\,dx$$



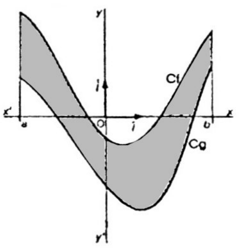
$$D = \left\{ M(x,y) \in \mathscr{P} \,/\, \begin{matrix} a \leqslant x \leqslant b \\ f(x) \leqslant y \leqslant 0 \end{matrix} \right\}$$

$$\mathscr{A}(D) = -\int_a^b f(x)\,dx$$



$$D = \left\{ M(x,y) \in \mathscr{P} \,/\, \begin{matrix} a \leqslant x \leqslant b \\ 0 \leqslant y \leqslant |f(x)| \end{matrix} \right\}$$

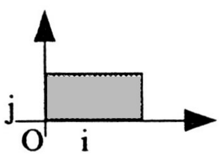
$$\mathscr{A}(D) = \int_a^c f(x)\,dx - \int_c^b f(x)\,dx$$



$$D = \left\{ M(x,y) \in \mathscr{P} \,/\, \begin{matrix} a \leqslant x \leqslant b \\ g(x) \leqslant y \leqslant f(x) \end{matrix} \right\}$$

$$\mathscr{A}(D) = \int_a^b [f(x)-g(x)]dx$$

unité d'aire (u.a.) = aire du rectangle unité :



Intégrale de Riemann

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left[\frac{b-a}{n}\sum_{k=0}^{n} f\left(a+k\frac{b-a}{n}\right)\right]$$